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Dissipative spherical collapse of charged anisotropic fluid in $f(R)$ gravity

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Abstract This manuscript is devoted to the study of the combined effect of a viable $f(R) = R + \alpha R^n$ model and the electromagnetic field on the instability range of gravitational collapse. We assume the presence of a charged anisotropic fluid that dissipates energy via heat flow and discuss how the electromagnetic field, density inhomogeneity, shear, and phase transition of astrophysical bodies can be incorporated by a locally anisotropic background. The dynamical equations help to investigate the evolution of self-gravitating objects and lead to the conclusion that the adiabatic index depends upon the electromagnetic background, mass, and radius of the spherical objects.

1 Introduction

Gravitational collapse is a highly dissipative phenomenon. The effects of dissipation describe a wide range of situations. For example, using a quasi-static approximation, limit cases of radiative transport have been studied in [1]. It is found that the hydrostatic time scale is very small as compared to the stellar lifetimes for different phases of a star's life. It is of the order of 27 min for the sun, 4.5 s for a white dwarf and 10^{-4} s for a neutron star of one solar mass and 10 km radius [2–4]. The dissipative factors enhance the instability range in the Newtonian limits but develop more stability in relativistic ranges. The impressions of radiation, anisotropy, and shearing viscosity at Newtonian and post-Newtonian eras are inquired in [5–8]. Various prospects of the collapse phenomenon to account for dark source have been worked out in the recent past [9–14]. Due to a high dissipation, matter produces a large amount of charge in the collapse phenomenon and so one is well motivated to investigate the effects of the electromagnetic field on the gravitational collapse [15].

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Chandrasekhar [16] took the initiative to work out the dynamical instability problem. Dynamical instability is significant in establishing the evolution and formation of stellar objects that must be stable against fluctuations. Generally, the adiabatic index Γ is useful to address the instability problem. The isotropic spheres of mass M and R radius may be related by $\Gamma \geq \frac{4}{3} + n\frac{M}{r}$, where the number n depends upon the star's structure. Later on the instability range for anisotropic, adiabatic, non-adiabatic, and shearing viscous fluids had been examined in [17–19]. Besides Γ, many other matter variables such as dissipation, radiation, shearing stress, anisotropy, expansion-free condition, etc. may also be responsible for the dynamical instability and the evolution in stars depending upon the properties of the fluid.

Modified theories of gravity have received enormous attention in recent years. The inclusion of higher order curvature invariants and coupled scalar fields has become a paradigm in alternative gravity theories. For this purpose, various alterations are made in the Einstein–Hilbert (EH) action [20–22]. The elementary and likely modification is to include curvature terms that are of type $f(R)$ having combinations of the Ricci scalar R. In this way, gravity tends to get modified on large scales, which reveals enormous observational signatures like a modified galaxy clustering spectrum [23,24], weak lensing [25,26], and the cosmic microwave background [27,28].

The most studied and simplest models in $f(R)$ theory are $f(R) = R + \sigma\frac{\mu^4}{R}$ and $f(R) = R + \alpha R^2$, where $\sigma = \pm 1, \alpha$ is a positive real number and μ is a parameter with the units of mass. Usually, positive values of the scalar curvature depicts standard cosmological corrections lead to de Sitter space [29], whereas negative values help to discuss the accelerating universe due to dark energy [30]. The effects of these $f(R)$ models on the dynamical instability of gravitational collapse has been discussed in recent papers [31,32]. In the same context, Sharif and Yousaf established the range of instability for a charged expansion-free, dissipative collapse for spherical and cylindrical symmetries in $f(R)$ gravity [33–35]. In this paper, we adopt $f(R)=R+\alpha R^n$ to discuss the dynamical instability of gravitational collapse in the background of an electromagnetic field.

The manuscript is arranged as follows. In Sect. 2, the energy-momentum tensor of the matter distribution along with Maxwell's and Einstein's field equations is given. Section 3 provides the knowledge about the adopted $f(R)$ model and the perturbation scheme. In the same section, the instability range will be discussed for Newtonian and post-Newtonian regimes in the form of Γ. Section 4 provides a summary of the paper and is followed by an appendix.

2 Evolution equations

We have chosen a timelike three dimensional spherical boundary surface Σ that delimits a four dimensional line element into two realms termed the exterior and interior regions. The interior region inside the boundary is

$$\mathrm{d}s_-^2 = A^2(t,r)\,\mathrm{d}t^2 - B^2(t,r)\,\mathrm{d}r^2 - C^2(t,r)(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2). \tag{2.1}$$

while line element for exterior region [35] is

$$\mathrm{d}s_+^2 = \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}\nu^2 + 2\,\mathrm{d}r\,\mathrm{d}\nu - r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2). \tag{2.2}$$

Here ν corresponds to the retarded time, M is the total mass, and Q indicates the total charge of the fluid.

The generalized EH action for $f(R)$ gravity to account for the Maxwell source is modified to

$$S=\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\frac{f(R)}{\kappa}-\frac{\digamma}{2\pi}\right). \tag{2.3}$$

Here κ stands for the coupling constant and $\digamma=\frac{1}{4}F^{uv}F_{uv}$ is the Maxwell invariant. We use the metric approach to recover the field equations by varying the above action with g_{uv} as follows:

$$f_R R_{uv} - \frac{1}{2}f(R)g_{uv} - \nabla_u\nabla_v f_R + g_{uv}\Box f_R = \kappa(T_{uv}+E_{uv}),\quad (u,v=0,1,2,3), \tag{2.4}$$

where $f_R \equiv \mathrm{d}f(R)/\mathrm{d}R$, ∇_u denotes the covariant derivative, $\Box=\nabla^u\nabla_v$, T_{uv} is the minimally coupled stress-energy tensor, and E_{uv} is the electromagnetic tensor. The above field equations can also be written as

$$G_{uv}=\frac{\kappa}{f_R}\left[L_{uv}\right], \tag{2.5}$$

where $L_{uv}=\overset{(D)}{T}_{uv}+T_{uv}+E_{uv}$ with

$$\overset{(D)}{T}_{uv}=\frac{1}{\kappa}\left[\frac{f(R)-Rf_R}{2}g_{uv}+\nabla_u\nabla_v f_R - g_{uv}\Box f_R\right] \tag{2.6}$$

denotes the effective stress-energy tensor. The usual matter is anisotropic and adiabatic in nature, representing dissipative collapse in the form of the heat flux q and is given by [17,36]

$$T_{uv}=(\mu+p_\perp)V_uV_v - p_\perp g_{uv} + (p_r-p_\perp)\chi_u\chi_v + q_uV_v + q_vV_u, \tag{2.7}$$

where μ stands for the density, p_r for the radial pressure, $p_\perp$ for the tangential pressure, V_u for the four-velocity of the fluid, and χ_u corresponds to the radial four vector. In co-moving coordinates, the following pattern is in order:

$$V^u=A^{-1}\delta^u_0,\quad q^u=qB^{-1}\delta^u_1,\quad \chi^u\chi_u=-1,\quad \chi^u=B^{-1}\delta^u_1. \tag{2.8}$$

The electromagnetic energy-momentum tensor is written as [37]

$$E_{uv}=\frac{1}{4\pi}\left(-F^w_u F_{vw}+\frac{1}{4\pi}F^{wx}F_{wx}g_{uv}\right). \tag{2.9}$$

Here $F_{uv}=\varphi_{v,u}-\varphi_{u,v}$ denotes the electromagnetic field tensor, while $\varphi_u=\varphi(t,r)\delta^0_u$ stands for the four potential. The Maxwell field equations are given by

$$F^{uv}_{;v}=\mu_0 J^u,\quad F_{uv;w}=0, \tag{2.10}$$

where $J^u=\mu(t,r)V^u$ is the four current, μ_0 is the magnetic permeability and μ represents the charge density. The electromagnetic field equations turn out to be

$$\frac{\partial^2\varphi}{\partial r^2}-\left(\frac{A'}{A}+\frac{B'}{B}-\frac{2C'}{C}\right)\frac{\partial\varphi}{\partial r}=4\pi\mu AB^2, \tag{2.11}$$

$$\frac{\partial^2\varphi}{\partial t\partial r}-\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{2\dot{C}}{C}\right)\frac{\partial\varphi}{\partial r}=0. \tag{2.12}$$

Herein derivatives with respect to t and r are expressed by a dot and a prime, respectively. Applying integration on Eq. (2.11), we have

$$\frac{\partial\varphi}{\partial r}=\frac{qBA}{C^2}. \tag{2.13}$$

The total charge q interior to the radius r with an electric field intensity E has the form

$$q=\int_0^r \mu BC^2\,\mathrm{d}r,\quad E=\frac{q}{4\pi C^2}. \tag{2.14}$$

For the interior spacetime, the components on the right hand side of the field equations (2.5) are given as follows, whereas the components of the Einstein tensor are present in [12]:

$$G_{00}=\frac{1}{f_R}\left\{\kappa\left(2\pi E^2+\rho\right)+\frac{f-Rf_R}{2}+\frac{f_R''}{B^2}\right.$$
$$\left.-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{B}}{B}+\frac{2\dot{C}}{C}\right)-\frac{f_R'}{B^2}\left(\frac{B'}{B}-\frac{2C'}{C}\right)\right\}, \quad (2.15)$$

$$G_{01}=\frac{\kappa}{2f_R}\left\{qAB+\frac{1}{\kappa}\left(\dot{f_R}'-\frac{A'}{A}\dot{f}_R-\frac{\dot{B}}{B}f_R'\right)\right\}, \quad (2.16)$$

$$G_{11}=\frac{1}{f_R}\left[\kappa\left(p_r-2\pi E^2\right)-\frac{f-Rf_R}{2}+\frac{\ddot{f}_R}{A^2}\right.$$
$$\left.-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}-\frac{2\dot{C}}{C}\right)-\frac{f_R'}{B^2}\left(\frac{B'}{B}+\frac{2C'}{C}\right)\right], \quad (2.17)$$

$$G_{22}=\frac{1}{f_R}\left[\kappa\left(p_\perp+2\pi E^2\right)-\frac{f-Rf_R}{2}+\frac{\ddot{f}_R}{A^2}\right.$$
$$-\frac{f_R''}{B^2}-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)$$
$$\left.-\frac{f_R'}{B^2}\left(\frac{A'}{A}-\frac{B'}{B}+\frac{C'}{C}\right)\right]. \quad (2.18)$$

The development in the collapsing phenomenon with the passage of time can be described by dynamical equations. These dynamical evolution equations for the usual matter, the effective, and the Maxwell energy-momentum tensor carrying higher order curvature invariants are formed by employing Bianchi identities as

$$L^{uv}_{;v}V_u=0,\quad L^{uv}_{;v}\chi_u=0, \quad (2.19)$$

which turn out to be

$$\dot{\rho}+q'\frac{A}{B}+2q\frac{A}{B}\left(\frac{A'}{A}+\frac{C'}{C}\right)+\rho\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)$$
$$+p_r\frac{\dot{B}}{B}+2p_\perp\frac{\dot{C}}{C}+4\pi E^2\left(\frac{\dot{E}}{E}+\frac{\dot{C}}{C}\right)+P_1(r,t)=0, \quad (2.20)$$

$$p_r'+p_r\left(\frac{A'}{A}+\frac{C'}{C}\right)+\rho\frac{A'}{A}-2p_\perp\frac{C'}{C}+\dot{q}\frac{B}{A}$$
$$+2\frac{B}{A}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-4\pi E^2\left(\frac{E'}{E}+\frac{2C'}{C}\right)+P_2(r,t)=0. \quad (2.21)$$

Here $P_1(r,t)$ and $P_2(r,t)$ corresponds to dark source terms provided in the appendix in the form of Eqs. (5.2) and (5.2), respectively.

3 $f(R)$ Model and perturbation scheme

The $f(R)$ model under consideration is

$$f(R)=R+\alpha R^n. \quad (3.1)$$

The configuration of second order derivative determines whether the model is viable or not. Any $f(R)$ is supposed to be suitable in GR and in the Newtonian limits if $f''(R)>0$. For our proposed form of $f(R)$, $n>2$, and α is a positive real number, in order to fulfill the stability criterion and demonstrate the accelerated expansion of the universe.

It is a fact that the general solution of the gravitational field equations is yet unknown because these are highly complicated non-linear differential equations. Perturbation theory can be employed to somehow incorporate these discrepancies, so that the dynamical equations become linear in terms of the metric and the material variables. Evolution can be explored by using Eulerian or Lagrangian schemes, i.e., by using fixed or co-moving coordinates, respectively. We have applied a perturbation with the assumption that initially all the metric and material functions are in static equilibrium, and with the passage of time perturbed quantities have both a radial and a time dependence. With $0<\varepsilon\ll 1$, the functions may be written in the following pattern:

$$A(t,r)=A_0(r)+\varepsilon T(t)a(r), \quad (3.2)$$
$$B(t,r)=B_0(r)+\varepsilon T(t)b(r), \quad (3.3)$$
$$C(t,r)=rB(t,r)[1+\varepsilon T(t)\bar{c}(r)], \quad (3.4)$$
$$\rho(t,r)=\rho_0(r)+\varepsilon\bar{\rho}(t,r), \quad (3.5)$$
$$p_r(t,r)=p_{r0}(r)+\varepsilon\bar{p}_r(t,r), \quad (3.6)$$
$$p_\perp(t,r)=p_{\perp 0}(r)+\varepsilon\bar{p_\perp}(t,r), \quad (3.7)$$
$$m(t,r)=m_0(r)+\varepsilon\bar{m}(t,r), \quad (3.8)$$
$$q(t,r)=\varepsilon\bar{q}(t,r), \quad (3.9)$$
$$R(t,r)=R_0(r)+\varepsilon T(t)e(r), \quad (3.10)$$
$$E(t,r)=E_0(r)+\varepsilon T(t)h(r) \quad (3.11)$$
$$f(R)=\left[R_0(r)(1+\alpha R_0^{n-1}(r))\right]$$
$$+\varepsilon T(t)e(r)\left[1+\alpha nR_0^{n-1}(r)\right], \quad (3.12)$$
$$f_R(R)=1+\alpha nR_0^{n-1}(r)+\varepsilon\alpha n(n-1)R_0^{n-2}(r)T(t)e(r). \quad (3.13)$$

Taking $C_0(r)=r$ as a Schwarzschild coordinate, the static configuration of the field equations (2.15)–(2.18) takes the following form:

$$\frac{2B_0'}{rB_0}+\frac{B_0^2-1}{r^2}=\frac{\kappa B_0^2}{1+\alpha nR_0^{n-1}}\left[\rho_0+2\pi E_0^2\right.$$
$$+\frac{\alpha n(n-1)R_0^{n-2}}{\kappa}\left\{-\frac{R_0^2}{2n}+\frac{(n-2)R_0^{-1}}{B_0^2}\right.$$
$$\left.\left.-\frac{1}{B_0^2}\left(\frac{B_0'}{B_0}-\frac{2}{r}\right)\right\}\right], \quad (3.14)$$

$$\frac{2A_0'}{rA_0}+\frac{B_0^2+1}{r^2}=\frac{\kappa B_0^2}{1+\alpha nR_0^{n-1}}\left[p_{r0}-2\pi E_0^2\right.$$
$$\left.+\frac{\alpha n(n-1)R_0^{n-2}}{\kappa}\left\{\frac{R_0^2}{2n}-\frac{1}{B_0^2}\left(\frac{A_0'}{A_0}-\frac{2}{r}\right)\right\}\right], \quad (3.15)$$

$$\frac{1}{r}\left(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}\right)+\frac{A_0''}{A_0}-\frac{A_0'B_0'}{A_0B_0}=\frac{\kappa B_0^2}{1+\alpha n R_0^{n-1}}\times\left[\frac{\alpha n(n-1)R_0^{n-2}}{\kappa}\left\{\frac{R_0}{2n}-\frac{(n-2)R_0^{-1}}{B_0^2}-\frac{1}{B_0^2}\left(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{1}{r}\right)\right\}+p_{\perp 0}+2\pi E_0^2\right]. \tag{3.16}$$

The first dynamical equation (2.20) is identically satisfied in the static configuration, while the second evolution equation (2.21) has a static configuration as under

$$p_r'+(\rho_0+p_{r0})\frac{A_0'}{A_0}+(p_{r0}-p_{\perp 0})\frac{2}{r}-4\pi E_0^2\left(\frac{E_0'}{E_0}+\frac{2}{r}\right)+P_{2s}=0, \tag{3.17}$$

where P_{2s} represents the static part of $P_2(r,t)$ and is provided in the appendix as Eq. (5.3). The perturbed configuration of the evolution equations (2.20) and (2.21) reads

$$\dot{\bar{\rho}}-\bar{q}'\frac{A_0}{B_0}+2\bar{q}\frac{A_0}{B_0}\left(\frac{A_0'}{A_0}+\frac{1}{r}\right)+2\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)+\dot{T}\left[\frac{b}{B_0}(\rho_0+p_{r0})+\frac{2\bar{c}}{r}(\rho_0+p_{\perp 0})+P_{1p}\right]=0 \tag{3.18}$$

$$\bar{p_r}'+\dot{\bar{q}}\frac{B_0}{A_0}+(\bar{\rho}+\bar{p_r})\frac{A_0'}{A_0}+(2\bar{p_r}+\bar{p_\perp})\frac{1}{r}+\left[-4\pi\left\{(E_0h)'+2E_0^2\left(\frac{\bar{c}}{r}\right)'+2E_0h\left(\frac{E_0'}{E_0}+\frac{2}{r}\right)\right\}+(\rho_0+p_{r0})\left(\frac{a}{A_0}\right)'+(2p_{r0}+p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right]T+P_{2p}=0, \tag{3.19}$$

where P_{1p} and P_{2p} denote the perturbed parts of P_1 and P_2, respectively, and are given in the appendix. Elimination of $\bar{q}$ from the perturbed equation (2.16) implies

$$\bar{q}=\frac{1}{\kappa A_0B_0}\left[\alpha n(n-1)R_0^{n-2}\left\{e'+e(n-2)R_0^{-1}R_0'-e\frac{A_0'}{A_0}-\frac{b}{B_0}R_0'\right\}-2(1+\alpha nR_0^{n-1})\times\left\{\frac{\bar{c}A_0'}{rA_0}+\frac{b}{rB_0}-\frac{\bar{c}'}{r}\right\}\right]\dot{T}. \tag{3.20}$$

On substitution of $\bar{q}$ and its radial derivative in Eq. (3.18) one is led to an equation from which $\dot{\bar{\rho}}$ can be extracted. Integrating this $\dot{\bar{\rho}}$ with respect to 't", we get

$$\bar{\rho}=\left[-\frac{b}{B_0}(\rho_0+p_{r0})-\frac{2\bar{c}}{r}(\rho_0+p_{\perp 0})-4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)+P_3(r)\right]T, \tag{3.21}$$

where $P_3(r)$ is presented in the appendix. By the second law of thermodynamics, $\bar{\rho}$ and $\bar{p}_r$ can be related as the ratio of specific heat with the assumption of a Harrison–Wheeler type equation of state, expressed in the following expression [8,38]:

$$\bar{p}_r=\Gamma\frac{p_{r0}}{\rho_0+p_{r0}}\bar{\rho}. \tag{3.22}$$

The adiabatic index Γ is a measure of the pressure variation with changing density. Putting Eq. (3.21) in the above equation, we arrive at

$$\bar{p}_r=-\Gamma\left[p_{r0}\frac{b}{B_0}+\frac{2\bar{c}}{r}\frac{p_{r0}(\rho_0+p_{\perp 0})}{\rho_0+p_{r0}}+4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)\frac{p_{r0}}{\rho_0+p_{r0}}-\frac{p_{r0}}{\rho_0+p_{r0}}P_3\right]T. \tag{3.23}$$

Insertion of $\bar{\rho}$, $\bar{q}$ and $\bar{p}_r$ from Eqs. (3.20), (3.21), and (3.23), respectively, in (3.19) leads to

$$\begin{aligned}&\frac{\ddot{T}}{\kappa A_0^2}\left[\alpha n(n-1)R_0^{n-2}\left\{e'+e(n-2)R_0^{-1}R_0'-e\frac{A_0'}{A_0}-\frac{b}{B_0}R_0'\right\}-2(1+\alpha nR_0^{n-1})\left\{\frac{\bar{c}A_0'}{rA_0}+\frac{b}{rB_0}-\frac{\bar{c}'}{r}\right\}\right]\\&-\Gamma T\left[p_{r0}\frac{b}{B_0}+\frac{2\bar{c}}{r}\frac{p_{r0}(\rho_0+p_{\perp 0})}{\rho_0+p_{r0}}+\left\{4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)-P_3\right\}\frac{p_{r0}}{\rho_0+p_{r0}}\right]_{,1}\\&-\Gamma T\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\left[p_{r0}\frac{b}{B_0}+\frac{2\bar{c}}{r}\frac{p_{r0}(\rho_0+p_{\perp 0})}{\rho_0+p_{r0}}+\left\{4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)-P_3\right\}\frac{p_{r0}}{\rho_0+p_{r0}}\right]+\frac{\bar{p}_\perp}{r}\\&-\frac{A_0'}{A_0}\left[-\frac{b}{B_0}(\rho_0+p_{r0})-\frac{2\bar{c}}{r}(\rho_0+p_{\perp 0})-4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)+P_3(r)\right]T\\&+\left[-4\pi\left\{(E_0h)'+2E_0^2\left(\frac{\bar{c}}{r}\right)'+2E_0h\left(\frac{E_0'}{E_0}+\frac{2}{r}\right)\right\}+(\rho_0+p_{r0})\left(\frac{a}{A_0}\right)'+(2p_{r0}+p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right]T+P_{2p}=0.\end{aligned} \tag{3.24}$$

When we perturb the Ricci scalar curvature, we obtain the following differential equation:

$$\ddot{T}(t)-P_4(r)T(t)=0. \tag{3.25}$$

$P_4(r)$ is given in the appendix. The P_4 terms are presumed in a way that one preserves positivity for the establishment of the instability range. Consequently, the solution of Eq. (3.25) is obtained as

$$T(t)=-e^{\sqrt{P_4}t}. \tag{3.26}$$

To estimate the instability range in the Newtonian and post-Newtonian limits, the above equation will be used in Eq. (3.24).

3.1 Newtonian regime

In this approximation, we assume that $\rho_0 \gg p_{r0}$, $\rho_0 \gg p_{\perp 0}$, and $A_0=1$, $B_0=1$. By substituting these values in Eq. (3.24), we find

$$\begin{aligned}&\frac{\ddot{T}}{\kappa}\Bigg[\alpha n(n-1)R_0^{n-2}\{e'+e(n-2)R_0^{-1}R_0'-bR_0'\}\\&-\frac{2(1+\alpha nR_0^{n-1})}{r}\left[b-\bar{c}'\right]\Bigg]\\&-\Gamma T\left[p_{r0}b+\frac{2\bar{c}}{r}p_{r0}\right]'-\Gamma T\frac{2}{r}\left[p_{r0}b+\frac{2\bar{c}}{r}p_{r0}\right]\\&+\frac{\bar{p}_{\perp(N)}}{r}+\left[(2p_{r0}+p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right.\\&\left.-4\pi\left\{(E_0h)'+2E_0^2\left(\frac{\bar{c}}{r}\right)'+2E_0h\left(\frac{E_0'}{E_0}+\frac{2}{r}\right)\right\}+\rho_0a'\right]T\\&+P_{2p(N)}=0.\end{aligned}\tag{3.27}$$

Here $P_{2p(N)}$ denotes the Newtonian regime terms of the perturbed second Bianchi identity. Inserting the value of T from Eq. (3.26) in the above equation and rearranging, we have

$$\Gamma<\frac{\rho_0a'-4\pi\left\{(E_0h)'+2E_0^2(\frac{\bar{c}}{r})'+2E_0h(\frac{E_0'}{E_0}+\frac{2}{r})\right\}+P_{2p(N)}+P_5}{\left[p_{r0}b+\frac{2\bar{c}}{r}p_{r0}\right]'+\frac{2p_{r0}}{r}\left[b+\frac{2\bar{c}}{r}\right]}, \tag{3.28}$$

where $P_5(r)$ is given in the appendix. It is worth mentioning here that the adiabatic index depends upon the electric field intensity, pressure components, energy density, and scalar curvature terms in this limit. Thus, a collapsing star would be unstable as long as inequality (3.28) holds.

3.1.1 Asymptotic behavior

The expression for Γ takes the following form when $\alpha\to 0$:

$$\Gamma<\frac{a'\rho_0-4\pi\left\{(E_0h)'+2E_0^2(\frac{\bar{c}}{r})'+2E_0h(\frac{E_0'}{E_0}+\frac{2}{r})\right\}-\frac{2P_4}{r\kappa}\left[b-\bar{c}'\right]}{\left[p_{r0}b+\frac{2\bar{c}}{r}p_{r0}\right]'+\frac{2p_{r0}}{r}\left[b+\frac{2\bar{c}}{r}\right]}. \tag{3.29}$$

This result represents the Einstein solution.

3.2 Post-Newtonian regime

Here, we analyze relativistic impressions up to $O(\frac{m_0}{r}+\frac{Q^2}{2r^2})$. In this approximation, we take

$$A_0=1-\frac{m_0}{r}+\frac{Q^2}{2r^2},\quad B_0=1+\frac{m_0}{r}-\frac{Q^2}{2r^2}, \tag{3.30}$$

$$\Rightarrow\frac{A_0'}{A_0}=\frac{2}{r}\frac{Q^2-rm_0}{2rm_0-2r^2-Q^2},\quad\frac{B_0'}{B_0}=\frac{2}{r}\frac{Q^2-rm_0}{2rm_0+2r^2-Q^2}. \tag{3.31}$$

On insertion of Eqs. (3.30) and (3.31) in (3.24), Γ reads

$$\Gamma<\frac{W+X+\frac{\bar{p}_{\perp(PN)}}{r}+P_{2(PN)}}{N'-\frac{2}{r}\frac{rm_0-2r^2}{2rm_0-2r^2-Q^2}N}, \tag{3.32}$$

where $P_{4(PN)}$, $\bar{p}_{\perp(PN)}$, and $P_{2(PN)}$ correspond to the PN regime terms of P_4, $\bar{p}_\perp$, and P_2, respectively. However, for W and X we have the following expressions:

$$\begin{aligned}W=&\frac{4r^4P_{4(PN)}}{\kappa(2r^2+2rm_0-Q^2)^2}\\&\times\Bigg[\alpha n(n-1)R_0^{n-2}\{e'+e(n-2)R_0^{-1}R_0'\}\\&-\frac{2}{r}\frac{Q^2-rm_0}{2rm_0-2r^2-Q^2}\\&\times\left\{e\alpha n(n-1)R_0^{n-2}+\frac{2\bar{c}}{r}(1+\alpha nR_0^{n-1})\right\}\\&-\frac{2r^2}{2r^2+2rm_0-Q^2}\left\{b\alpha n(n-1)R_0^{n-2}R_0'\right.\\&\left.+\frac{2b}{r}(1+\alpha nR_0^{n-1})\right\}+\frac{2\bar{c}'}{r}(1+\alpha nR_0^{n-1})\Bigg],\end{aligned}\tag{3.33}$$

$$\begin{aligned}X=&-\frac{2}{r}\frac{Q^2-rm_0}{2rm_0-2r^2-Q^2}\left[\frac{2br^2(\rho_0+p_r)}{2rm_0+2r^2-Q^2}\right.\\&\left.-\frac{2\bar{c}}{r}(\rho_0+p_\perp)+P_{4(PN)}-4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)\right]\\&+4\pi\left\{(E_0h)'+2E_0^2\left(\frac{\bar{c}}{r}\right)'+2E_0h\left(\frac{E_0'}{E_0}+\frac{2}{r}\right)\right\}\\&-(\rho_0+p_r)\left(\frac{2ar^2}{2rm_0-2r^2+Q^2}\right)'+V(2p_r+p_\perp)\left(\frac{\bar{c}}{r}\right)',\end{aligned}\tag{3.34}$$

$$\begin{aligned}N=&\frac{2br^2p_r}{2rm_0+2r^2-Q^2}+\frac{p_r}{\rho_0+p_r}\left[4\pi E_0^2\left(\frac{h}{E_0}+\frac{2\bar{c}}{r}\right)\right.\\&\left.-P_{4(PN)}\right]+\frac{2\bar{c}}{r}\frac{p_r(\rho_0+p_\perp)}{\rho_0+p_r}.\end{aligned}\tag{3.35}$$

As far as the instability problem is concerned, the system is unstable in the PN limit as regards the above inequality. We can see how the curvature terms along with material variables affect the instability range. All terms appearing in the above inequality must maintain positivity to fulfill the dynamical instability condition, and hence we have the following constraints:

$$N' > \frac{2}{r}\frac{rm_0 - 2r^2}{2rm_0 - 2r^2 - Q^2}N,$$
$$3rm_0 > 2(r^2 + Q^2), \quad r^2 < Q^2 - 2rm_0.$$

3.2.1 Asymptotic behavior

As $\alpha \to 0$, the adiabatic index Γ is unchanged, whereas W becomes

$$W = \frac{4r^4}{\kappa(2rm_0 + 2r^2 - Q^2)^2(2rm_0 - 2r^2 - Q^2)} \times \left[\frac{2\bar{c}}{r^2}(Q^2 - rm_0) - 4br + \frac{2\bar{c}'}{r}\right]. \tag{3.36}$$

This represents the GR solution in the PN regime.

4 Summary and discussion

The purpose of the present work is to determine the electromagnetic field impressions on the instability of a spherically symmetric collapsing compact object in the $f(R)$ framework. In order to achieve this goal, locally anisotropic matter experiencing a dissipative collapse has been considered. We employ a Jordan frame to work out the instability problem and to modify the EH action for modified gravity; we consider $f(R) = R + \alpha R^n$ and a Maxwell source so that attribution of the $f(R)$ model along with an electromagnetic field can be investigated. The Einstein field equations are modified accordingly. Dynamical equations are developed to study the evolution of a non-static spherical star with the help of a perturbation approach.

The model under consideration provides a viable alternative to dark energy; it satisfies the condition $f''(R) > 0$ to realize a stellar stable configuration for the matter dominated regime. Dissipation in terms of heat flow plays an important role in the dynamics of collapse; especially the electric charge and its distribution imply drastic effects on the stellar evolution and structure.

A perturbed form of the second Bianchi describes the evolution of the collapsing system and is further used to establish the instability range in terms of the adiabatic index Γ. It is evident from the results that Γ is dependent on the electric field intensity, radiative effects, density, and pressure configuration. The inclusion of a Maxwell source along with higher order curvature invariants imply that the self-gravitating system becomes more stable in the presence of an electromagnetic field. The results are reduced to GR as $\alpha \to 0$.

Lastly, we compare our findings with the previous literature and find that our results reduce to the work already done for various constraints on n and electromagnetic effects. A comparison is elaborated in the following:

- For $p_r = p_\perp, n = -1$ and $\alpha = \delta^4$ our results reduce to the isotropic pressure case [34].
- In the absence of a Maxwell source the results correspond to the results presented in [31].
- When we take $n = 2$ in our model, the results support the arguments in [35]. Also, it is clear that the addition of a Maxwell invariant describes a more general expanding universe with a wider range of instability in the $f(R)$ framework.

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5 Appendix

We have

$$P_1(r,t) = \frac{1}{\kappa}\left[A^2\left\{\frac{1}{A^2}\left(\frac{f - Rf_R}{2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C}\right)\right.\right.\right.$$
$$\left.\left. - \frac{f_R'}{B^2}\left(\frac{B'}{B} - \frac{2C'}{C}\right) + \frac{f_R''}{B^2}\right)\right\}_{,0} + A^2\left\{\frac{1}{A^2B^2}\left(\dot{f}_R'\right.\right.$$
$$\left.\left. - \frac{A'}{A}\dot{f}_R - \frac{\dot{B}}{B}f_R'\right)\right\}_{,1} - \frac{\dot{f}_R}{A^2}\left\{\left(\frac{3A'}{A} + \frac{B'}{B} + \frac{2C'}{C}\right)\frac{AA'}{B^2}\right.$$
$$\left. + \left(\frac{\dot{B}}{B}\right)^2 + 2\left(\frac{\dot{C}}{C}\right)^2 + \frac{3\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C}\right)\right\}$$
$$+ \frac{\dot{f}_R'}{B^2}\left(\frac{3A'}{A} + \frac{B'}{B} + \frac{2C'}{C}\right) - \frac{2f_R'}{B^2}\left\{\frac{A'}{A}\left(\frac{2\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right.$$
$$\left. + \frac{B'}{B}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) - \frac{C'}{C}\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right\}$$
$$\left. + \frac{f_R''}{B^2}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\dot{A}}{A}(f - Rf_R) + \frac{\ddot{f}_R}{A^2}\left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C}\right)\right], \tag{5.1}$$

$$P_2(r,t) = \frac{1}{\kappa}\left[B^2\left\{\frac{1}{B^2}\left(\frac{Rf_R - f}{2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} - \frac{2\dot{C}}{C}\right)\right.\right.\right.$$
$$\left.\left. - \frac{f_R'}{B^2}\left(\frac{A'}{A} + \frac{2C'}{C}\right) + \frac{\ddot{f}_R}{A^2}\right)\right\}_{,1} + B^2\left\{\frac{1}{A^2B^2}\left(\dot{f}_R{}'\right.\right.$$
$$\left.\left. - \frac{A'}{A}\dot{f}_R - \frac{\dot{B}}{B}f_R'\right)\right\}_{,0} + \frac{A'}{A}\left\{\frac{\ddot{f}_R}{A^2} + \frac{f_R''}{B^2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right.$$
$$\left. - \frac{f_R'}{B^2}\left(\frac{A'}{A} + \frac{B'}{B}\right)\right\} + \frac{2B'}{B}\left\{\frac{Rf_R - f}{2} + \frac{\ddot{f}_R}{A^2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}\right.\right.$$
$$\left.\left. - \frac{2\dot{C}}{C}\right) - \frac{f_R'}{B^2}\left(\frac{A'}{A} + \frac{3C'}{C}\right)\right\} + \frac{1}{A^2}\left(\frac{\dot{A}}{A} + \frac{3\dot{B}}{B} + \frac{2\dot{C}}{C}\right)$$

$$\times\left(\dot{f_R}'-\frac{A'}{A}\dot{f_R}-\frac{\dot{B}}{B}f_R'\right)+\frac{2C'}{C}\times\left\{\frac{f_R''}{B^2}+\frac{\dot{f}_R}{A^2}\left(\frac{\dot{C}}{C}-\frac{2\dot{B}}{B}\right)-\frac{f_R'}{B^2}\frac{C'}{C}\right\}\Bigg]. \quad (5.2)$$

The static part of $P_2(r,t)$ is

$$P_{2s}=\frac{\alpha n(n-1)}{\kappa}\left[B_0^2\left\{\frac{R_0^{n-2}}{nB_0^2}\left(\frac{R_0^2}{2}-\frac{nR_0'}{B_0^2}\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\right)\right\}_{,1}+\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\left(\frac{R_0''+(n-2)R_0^{-1}{R_0'}^2}{B_0^2}\right)+\frac{2B_0'R_0^2}{2nB_0}\times\frac{R_0^{n-2}R_0'}{B_0^2}\left\{\frac{A_0'}{A_0}\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right)++\frac{2B_0'}{B_0}\left(\frac{A_0'}{A_0}+\frac{3}{r}\right)+\frac{2}{r^2}\right\}\right]. \quad (5.3)$$

The perturbed forms of P_1 and P_2 read

$$P_{1p}=\frac{e''+\alpha n(n-1)R_0^{n-2}}{\kappa B_0^2}\Bigg[(n-2)\left\{(2e'-e)R_0^{-1}R_0'+eR_0^{-1}R_0''+e(n-3)R_0^{-2}{R_0'}^2\right\}-\frac{R_0B_0^2}{2}-\frac{b}{B_0}(R_0''+(n-2)R_0^{-1}{R_0'}^2)+e'+R_0'\left\{\frac{\bar{c}}{r^2}+\frac{b'}{B_0}+\frac{2\bar{c}'}{r}-\frac{b}{B_0}\times\left(\frac{2A_0'}{A_0}+\frac{3B_0'}{B_0}+\frac{4}{r}-1\right)-\frac{2\bar{c}}{r}\left(\frac{A_0'}{A_0}-\frac{2}{r}\right)\right\}+\left(e'-e\frac{A_0'}{A_0}+(n-2)eR_0^{-1}R_0'\right)\left(\frac{3A_0'}{A_0}+\frac{B_0'}{B_0}+\frac{1}{r}\right)\Bigg]+\frac{\alpha n(n-1)}{\kappa}\frac{A_0^2}{B_0^2}\left[\frac{R_0^{n-2}}{A_0^2B_0^2}\left\{e'-e\frac{A_0'}{A_0}-\frac{b}{B_0}R_0'+(n-2)eR_0^{-1}R_0'\right\}\right]_{,1} \quad (5.4)$$

$$P_{2p}=\frac{\alpha n(n-1)}{\kappa}\Bigg[\frac{R_0^{n-2}}{A_0^2}\ddot{T}\left\{(e'+(n-2)eR_0^{-1}R_0')(1+A_0^2)-\frac{b}{B_0}R_0'+2e(1-A_0^2)\frac{B_0'}{B_0}\right\}+2TbB_0\left\{\frac{R_0^{n-2}}{B_0^4}\left\{R_0'\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)-\frac{R_0^2B_0^2}{2n}\right\}\right\}_{,1}+TB_0^2\left\{\frac{R_0^{n-2}}{B_0^4}\left[e\frac{R_0B_0^2}{2}+\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\left\{e(n-2)R_0^{-1}R_0'+\frac{R_0^2bB_0}{n}-R_0'\left[\left(\frac{a}{A_0}\right)'+2\left(\frac{\bar{c}}{r}\right)'+\frac{4b}{B_0}\right]+e'\right\}\right]\right\}_{,1}+\frac{R_0^{n-2}}{B_0^2}T\left\{\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\left\{e''+\left[2e'R_0^{-1}R_0'+eR_0^{-1}R_0''+e(n-3)R_0^{-2}{R_0'}^2\right](n-2)\right\}-\left\{2\left(\frac{a}{A_0}\right)'\left(\frac{\bar{c}}{r}\right)'+\frac{2b}{B_0}\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\right\}\times\left[R_0''+(n-2)R_0^{-1}{R_0'}^2\right]+R_0'\left\{\left(\frac{a}{A_0}\right)'\times\left(\frac{2A_0'}{A_0}+\frac{3B_0'}{B_0}\right)+3\left(\frac{b}{B_0}\right)'\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)+2\left(\frac{\bar{c}}{r}\right)'\left(\frac{3B_0'}{B_0}+\frac{2}{r}\right)\right\}+\left[e'+e(n-2)R_0^{-1}R_0'-\frac{2b}{B_0}R_0'\right]\left[\frac{A_0'}{A_0}\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right)+\frac{2}{r}\left(\frac{3B_0'}{B_0}+\frac{1}{r}\right)\right]\right\}\Bigg]=0 \quad (5.5)$$

The expressions for P_3 and P_4, respectively, are

$$P_3=-\frac{A_0}{B_0}\left[\frac{1}{\kappa A_0B_0}\left\{\alpha n(n-1)R_0^{n-2}\left(e'+e(n-2)R_0^{-1}R_0'-e\frac{A_0'}{A_0}-\frac{b}{B_0}R_0'\right)-2(1+\alpha nR_0^{n-1})\left(\frac{\bar{c}A_0'}{rA_0}+\frac{b}{rB_0}-\frac{\bar{c}'}{r}\right)\right\}\right]_{,1}-\frac{2}{\kappa B_0^2}\times\left[\alpha n(n-1)R_0^{n-2}\left(e'+e(n-2)R_0^{-1}R_0'-e\frac{A_0'}{A_0}-\frac{b}{B_0}R_0'\right)-2(1+\alpha nR_0^{n-1})\left(\frac{\bar{c}A_0'}{rA_0}+\frac{b}{rB_0}-\frac{\bar{c}'}{r}\right)\right]\left(\frac{A_0'}{A_0}+\frac{1}{r}\right)-P_{1p} \quad (5.6)$$

$$P_4=-\frac{rA_0^2B_0}{br+2B_0\bar{c}}\left[\frac{e}{2}-\frac{2\bar{c}}{r^3}-\frac{1}{A_0B_0^2}\left\{A_0''\left[\frac{a}{A_0}+\frac{2b}{B_0}\right]-\frac{1}{B_0}\left(a'B_0'+a''+A_0'b'-A_0'B_0'\left[\frac{a}{A_0}+\frac{3b}{B_0}\right]\right)+\frac{2}{r}\left\{a'+\bar{c}'A_0'-A_0'\left[\frac{a}{A_0}+\frac{2b}{B_0}+\frac{\bar{c}}{r}\right]\right\}+\frac{A_0}{r}\left\{\bar{c}''-\frac{b'}{B_0}-\frac{B_0'\bar{c}'}{B_0}+\frac{3b}{B_0}+\frac{\bar{c}}{r}\right\}+\frac{2}{r^2}\left[\bar{c}'-\frac{b}{B_0}-\frac{\bar{c}}{r}\right]\right\}\right]=0. \quad (5.7)$$

$$P_5=\frac{P_4}{\kappa}\left[\alpha n(n-1)R_0^{n-2}\{e'+e(n-2)R_0^{-1}-bR_0'\}-\frac{2(1+\alpha nR_0^{n-1})}{r}\left[b-\bar{c}'\right]\right] \quad (5.8)$$

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